Two-flux Colliding Plane Waves in String Theory

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Abstract

We construct the two-flux colliding plane wave solutions in higher dimensional gravity theory with dilaton, and two complementary fluxes. Two kinds of solutions has been obtained: Bell-Szekeres(BS) type and homogeneous type. After imposing the junction condition, we find that only Bell-Szekeres type solution is physically well-defined. Furthermore, we show that the future curvature singularity is always developed for our solutions.

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1 Introduction

The gravitational colliding plane wave (CPW) in four dimensional gravity has been well studied since its discovery in the early 1970s[1, 2]. (For a complete review on four dimensional CPW solutions, see [3]) The CPW solutions are the exact classical solutions of the Einstein gravity, describing the collision of the plane waves. It is remarkable that a late time scalar curvature singularity always develop in the CPW solutions, indicating the fact that the two colliding plane gravitational wave focus each other so strongly as to produce a spacetime singularity[4, 5]. It's true that there exist infinite families of solutions with a horizon rather than a curvature singularity. However, such solutions are unstable with respect to small perturbations in the initial data. Generically, a curvature singularity forms[5]. Therefore, it is hoped that the study of the CPW solutions may help us to have a better understanding of the singularity. Also, it has been proposed that the gravitational plane and dilatonic waves could play an important role in the pre-big-bang cosmology scenarios[6].

Very recently, there are much interests in the study of CPW solutions in the higher dimensional gravity. On one side, the higher dimensional gravity has turned out to have much more interesting properties than four dimensional gravity. One example is that the uniqueness and stability issue in higher dimensional black holes are more complicated[7]. It has been found that there exist *black ring* solution with horizon topology $S^1 \times S^2$ in five dimensional gravity[8]. It could be wished that the CPW solutions in the higher dimensional gravity have more rich structures and physics. On the other hand, the plane waves is not only the classical solutions to vacuum Einstein equation but also the one to string theory[9]. It is quite interesting to study their collision in the framework of low energy effective action of string theory. It is well known that in the low energy effective action, there are dilaton fields and various kinds of multi-form fields, coupled with each other in the supergravity action. Obviously, the complete and thorough discussion of CPW solutions in higher dimensional gravity is still out of our reach. Nevertheless, there have been some progress along this direction. In [10], Gurses et.al discussed the CPW solutions in dilaton gravity, in higher dimensional gravity, and in higher dimensional Einstein-Maxwell theory. In [11], Gutperle and Pioline tried to construct the CPW solutions in the ten-dimensional gravity with self-dual form flux. However, their generalized BS-type solution (3.37) fails to satisfy the junction condition and is unphysical. Only in [13], the flux-CPW solution, which named for the CPW solution with flux, has been successfully constructed in a higher dimensional gravity theory with dilaton and a higher form flux. Actually, There are two classes of solutions: one is called $(pqrw)$-type. It looks like the Bell-Szekeres solutions, which describe the electric-magnetic CPW in four-dimensional Einstein-Maxwell gravity[12]. The other class is a new kind of homogeneous solution, called $(f \pm g)$ type.

In this paper, we would like to generalize the study in [13] to the case with two complementary fluxes in the theory. We manage to solve the equation of motions, which is more involved than the ones in [13] and also find two classes of solutions generically. But after imposing the junction conditions, we find that only BS-type solution is physically well defined and acceptable while the $(f \pm g)$ type solution fails to satisfy the junction conditions. We also notice that the future singularity will always be developed in our solutions.

The paper is organized as follows: in section two, after giving a brief review on how to find the CPW solution in general, we derive the equations of motions and solve them; in section three, we impose the junction condition on the solutions and study the future singularity of the solution; in section four, we end with some conclusions and discussions. To be self-consistent, we include the Riemann and Ricci tensor for our metric ansatz in the appendix.

2 Two-flux-CPW solutions to the equations of motions

In the study of the collision of the gravitational plane waves, one usually divides the spacetime into four regions: past P-region($u < 0, v < 0$), right R-region($u > 0, v < 0$), left L-region($u < 0, v > 0$) and future F-region($u > 0, v > 0$), which describes the Minkowski spacetime, the incoming waves from right and left, and the colliding region respectively. The general recipe to construct the CPW solutions is to solve

the equations of motions in the forward region and then reduce the solutions to other regions, requiring the metric to be continuous and invertible in order to paste the solutions in different regions. More importantly, one need to impose the junction conditions to get an acceptable physical solution. In this section, we focus on the solutions to the equations of motions and leave the discussion on the junction conditions to the next section.

We will work in a higher dimensional gravity theory with dilaton and two complementary fluxes. More precisely, let us consider the following action with the graviton, the dilaton, a $(n+1)$-form and a $(m+1)$-form field strength in $D(=n+m+2)$-dimensional space-time

$$S=\int d^D x\sqrt{-g}\left(R-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{1}{2(n+1)!}e^{a\phi}F^2-\frac{1}{2(m+1)!}e^{b\phi}H^2\right) \tag{1}$$

where $F^2=F_{\mu_1\ldots\mu_{n+1}}F^{\mu_1\cdots\mu_{n+1}}, H^2=H_{\nu_1\ldots\nu_{m+1}}H^{\nu_1\ldots\nu_{m+1}}$ and a,b are the dilaton coupling constants. Such kind of action could often appear as the bosonic part of $D>4$ supergravity theories, which could be the low energy effective actions of string theory in the Einstein frame or their Kaluza-Klein reduction. For example, in 5-dim supergravity, one may have dilaton, gauge 1-form field, and also NS $B_{\mu\nu}$ 2-form field; in 6-dim supergravity from Kaluza-Klein reduction of 10-dim supergravity, one may have dilaton, gauge 1-form field and 3-form field; even in 10-dim supergravity, if m=n=4, our case could be reduced to the one in Gutperle and Pioline's paper. Usually, the field content of the $D>4$ supergravity has more fields and Chern-Simons coupling. After turning off the extra fields and making the ansatz of the flux field strength to make the Chern-Simons coupling vanishing, the action could reduce to (1).

From (1), the equations of motions are given by

$$\begin{aligned}R_{\mu\nu}=\partial_\mu\phi\partial_\nu\phi+\frac{1}{2n!}e^{a\phi}\left(F_{\mu\mu_1\ldots\mu_n}F_\nu{}^{\mu_1\ldots\mu_n}-\frac{n}{(n+1)(m+n)}g_{\mu\nu}F^2\right)+\\+\frac{1}{2m!}e^{b\phi}\left(H_{\mu\nu_1\ldots\nu_m}H_\nu{}^{\nu_1\ldots\nu_m}-\frac{m}{(m+1)(m+n)}g_{\mu\nu}H^2\right)\end{aligned} \tag{2}$$

$$\partial_\mu\left(\sqrt{-g}e^{a\phi}F^{\mu\mu_1\ldots\mu_n}\right)=0 \tag{3}$$

$$\partial_\nu\left(\sqrt{-g}e^{b\phi}H^{\nu\nu_1\ldots\nu_m}\right)=0 \tag{4}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right)=\frac{a}{4(n+1)!}e^{a\phi}F^2+\frac{b}{4(m+1)!}e^{b\phi}H^2 \tag{5}$$

We make the following CPW ansatz for the metric

$$d^2s=2e^{-M}dudv+e^A\sum_{i=1}^n dx_i^2+e^B\sum_{j=1}^m dy_j^2, \tag{6}$$

and to keep problem tractable, we also restrict ourselves to the case that the nonzero components of the $(n+1)$-form,$(m+1)$-form fluxes are

$$\begin{aligned}F_{ux_1\ldots x_n}&=C_u &\qquad F_{vx_1\ldots x_n}&=C_v\\ H_{uy_1\ldots y_n}&=D_u &\qquad H_{vy_1\ldots y_n}&=D_v\end{aligned} \tag{7}$$

In the sense that the two fluxes F and H occupy mostly the x_i's and y_j's respectively, we regard them to be complementary. We will take the functions M,A,B,C,D and ϕ to be function of u,v only. The equations of motions for the graviton take the form

$$nA_{uu}+mB_{uu}+nM_uA_u+mM_uB_u+\frac{1}{2}(nA_u^2+mB_u^2)=-2\phi_u^2-e^{a\phi-nA}C_u^2-e^{b\phi-mB}D_u^2 \tag{8}$$

$$nA_{vv}+mB_{vv}+nM_vA_v+mM_vB_v+\frac{1}{2}(nA_v^2+mB_v^2)=-2\phi_v^2-e^{a\phi-nA}C_v^2-e^{b\phi-mB}D_v^2 \tag{9}$$

$$-M_{uv}+\frac{n}{2}A_{uv}+\frac{m}{2}B_{uv}+\frac{1}{4}(nA_uA_v+mB_uB_v)=-\phi_u\phi_v+\frac{n-m}{2(n+m)}e^{a\phi-nA}C_uC_v+\frac{m-n}{2(n+m)}e^{b\phi-mB}D_uD_v \tag{10}$$

$$2A_{uv}+nA_uA_v+\frac{m}{2}(A_uB_v+A_vB_u)=-\frac{2m}{n+m}e^{a\phi-nA}C_uC_v+\frac{2m}{n+m}e^{b\phi-mB}D_uD_v \tag{11}$$

$$2B_{uv}+mB_uB_v+\frac{n}{2}(A_uB_v+A_vB_u)=\frac{2n}{n+m}e^{a\phi-nA}C_uC_v-\frac{2n}{n+m}e^{b\phi-mB}D_uD_v \tag{12}$$

The equations of motions for the dilaton and n-form, m-form potential are given by

$$2C_{uv}+\left[a\phi-\frac{1}{2}(nA-mB)\right]_u C_v+\left[a\phi-\frac{1}{2}(nA-mB)\right]_v C_u=0 \tag{13}$$

$$2D_{uv}+\left[b\phi+\frac{1}{2}(nA-mB)\right]_u D_v+\left[b\phi+\frac{1}{2}(nA-mB)\right]_v D_u=0 \tag{14}$$

$$\phi_{uv}+\frac{1}{4}(nA+mB)_u\phi_v+\frac{1}{4}(nA+mB)_v\phi_u=\frac{a}{4}e^{a\phi-nA}C_uC_v+\frac{b}{4}e^{b\phi-mB}D_uD_v \tag{15}$$

Here we have abbreviated the derivatives by a subscript, e.g. $A_u=\partial_uA$. As usual, the equation (10) is redundant and will not be needed anymore. Introduce

$$U=\frac{1}{2}(nA+mB) \qquad\qquad V=\frac{1}{2}(nA-mB) \tag{16}$$

to make the equations (8), (9), (11), (12) become

$$U_{uu}+M_uU_u+\frac{m+n}{4mn}\left(U_u^2+V_u^2\right)+\frac{m-n}{2mn}U_uV_u=-\phi_u^2-\frac{1}{2}e^{a\phi-nA}C_u^2-\frac{1}{2}e^{b\phi-mB}D_u^2 \tag{17}$$

$$U_{vv}+M_vU_v+\frac{m+n}{4mn}\left(U_v^2+V_v^2\right)+\frac{m-n}{2mn}U_vV_v=-\phi_v^2-\frac{1}{2}e^{a\phi-nA}C_v^2-\frac{1}{2}e^{b\phi-mB}D_v^2 \tag{18}$$

$$U_{uv}+U_uU_v=0 \tag{19}$$

$$V_{uv}+\frac{1}{2}(U_uV_v+U_vV_u)=-\frac{mn}{m+n}e^{a\phi-nA}C_uC_v+\frac{mn}{m+n}e^{b\phi-mB}D_uD_v \tag{20}$$

Equation (19) is well-known in the study of CPW solutions, and the general solution to it is

$$U=\log\left[f(u)+g(v)\right] \tag{21}$$

where f,g are arbitrary functions, chosen usually to be monotonic functions. It is convenient to treat (f,g) as coordinates alternative to (u,v).

2.1 Two-flux-CPW solutions to equations of motions: when $a\neq -b$

In the case that $a\neq -b$, one can define

$$X=a\phi-V \qquad\qquad Y=b\phi+V \tag{22}$$

to simplify the equations (13), (14) (15) and (20) in terms of the (f,g)-coordinates. After some linear combinations, one has:

$$(f+g)X_{fg}+\frac{1}{2}X_f+\frac{1}{2}X_g=\frac{1+\delta a^2}{4\delta}e^XC_fC_g-\frac{1-\delta ab}{4\delta}e^YD_fDg \tag{23}$$

$$(f+g)Y_{fg}+\frac{1}{2}Y_f+\frac{1}{2}Y_g=-\frac{1-\delta ab}{4\delta}e^XC_fC_g+\frac{1+\delta b^2}{4\delta}e^YD_fD_g \tag{24}$$

$$2C_{fg}+X_fC_g+X_gC_f=0 \tag{25}$$
$$2D_{fg}+Y_fD_g+Y_gD_f=0 \tag{26}$$

where
$$\delta:=\frac{m+n}{4mn}\le\frac{1}{2}. \tag{27}$$

In terms of the (f,g)-coordinates, the equations (17) and (18) can be written as

$$S_f+\frac{1}{2}e^XC_f^2+\frac{1}{2}e^YD_f^2+\frac{(f+g)}{(a+b)^2}\left[(1+\delta a^2)Y_f^2+(1+\delta b^2)X_f^2+2(1-\delta ab)X_fY_f\right]=0 \tag{28}$$

$$S_g+\frac{1}{2}e^XC_g^2+\frac{1}{2}e^YD_g^2+\frac{(f+g)}{(a+b)^2}\left[(1+\delta a^2)Y_g^2+(1+\delta b^2)X_g^2+2(1-\delta ab)X_gY_g\right]=0 \tag{29}$$

where
$$S=M-(1-\delta)\log(f+g)+\log(f_ug_v)+\eta V \tag{30}$$

and
$$\eta:=\frac{m-n}{2mn} \tag{31}$$

The inverse relation of (22) is

$$V=\frac{aY-bX}{a+b}. \qquad \phi=\frac{X+Y}{a+b} \tag{32}$$

Our strategy here is to solve the above set of coupled differential equations of (S,X,Y,C,D) as the functions of (f,g) and then get (M,A,B,C,D,ϕ) by straightforward derivation. As the first step, we need to solve the coupled differential equations of (X,Y,C,D). Though they seem to be more complicated than the analogue ones in [13], we manage to find two kinds of solutions with two different forms of ansatz.

- (pqrw)-type (BS type) solution:

  We may try the following ansatz for X,Y,C,D.

$$X=-\log c_1\frac{rw+pq}{rw-pq} \qquad Y=-\log c_2\frac{rw+pq}{rw-pq}$$
$$C=\gamma_1(pw-rq) \qquad D=\gamma_2(pw-rq) \tag{33}$$

  where
$$p:=\sqrt{\frac{1}{2}-f} \qquad q:=\sqrt{\frac{1}{2}-g} \qquad r:=\sqrt{\frac{1}{2}+f} \qquad w:=\sqrt{\frac{1}{2}+g}. \tag{34}$$

  They satisfy the equations (25) and (26) automatically and from (23) (24), we have

$$\gamma_1^2=\frac{8(2+\delta b^2-\delta ab)c_1}{(a+b)^2} \tag{35}$$

$$\gamma_2^2=\frac{8(2+\delta a^2-\delta ab)c_2}{(a+b)^2} \tag{36}$$

  After integrating (28) and (29) with respect to f and g, we find that

$$S=\frac{4+\delta(a-b)^2}{(a+b)^2}\left[\log(1-2f)(1+2g)+\log(1+2f)(1-2g)-\log(f+g)\right] \tag{37}$$

and then

$$e^{-M} = c_1^{\frac{b}{a+b}\eta} c_2^{\frac{-a}{a+b}\eta} f_u g_v \left[(1-4f^2)(1-4g^2)\right]^{-\frac{4+\delta(a-b)^2}{(a+b)^2}} (f+g)^{-\left[1-\delta-\frac{4+\delta(a-b)^2}{(a+b)^2}\right]} \left(\frac{rw+pq}{rw-pq}\right)^{\frac{b-a}{a+b}\eta} \tag{38}$$

$$e^{nA} = c_1^{\frac{b}{a+b}} c_2^{\frac{-a}{a+b}} (f+g) \left(\frac{rw+pq}{rw-pq}\right)^{\frac{b-a}{a+b}} \tag{39}$$

$$e^{mB} = c_1^{\frac{-b}{a+b}} c_2^{\frac{a}{a+b}} (f+g) \left(\frac{rw+pq}{rw-pq}\right)^{\frac{a-b}{a+b}} \tag{40}$$

and also the dilaton field is given by

$$e^{\phi} = (c_1 c_2)^{\frac{-1}{a+b}} \left(\frac{rw+pq}{rw-pq}\right)^{\frac{-2}{a+b}} \tag{41}$$

Up to now, we have solved the equations of motions in the F-region and find a two-parameter family of solutions depending on the constants c_1 and c_2. Actually one can reduce the above solutions to the ones for the L-region, the R-region, and the P-region if one do the following replacements:

$$f(u) = f_0 \qquad f_u(1-2f)^{-\rho}\,|_{f=f_0} = -1 \qquad \text{for} \quad u<0 \tag{42}$$

$$g(v) = g_0 \qquad g_v(1-2g)^{-\rho}\,|_{g=g_0} = -1 \qquad \text{for} \quad v<0 \tag{43}$$

where $\rho = \frac{4+\delta(a-b)^2}{(a+b)^2}$ and f_0, g_0 are constants. Taking into account of the continuous and invertible conditions on the metric, we are able to fix the values of

$$f_0 = g_0 = 1/2 \tag{44}$$

and put constraints on the parameters c_1, c_2:

$$c_1^b = c_2^a. \tag{45}$$

Finally, we only have an one-parameter family of $(pqrw)$-type solutions.

- $(f \pm g)$-type solution

  For the second type solution, we take the following ansatz for X, Y, C and D whose dependence in f, g are of the form $(f \pm g)$:

$$\begin{aligned} X &= X(f+g) & \qquad Y &= Y(f+g) \\ C &= C(f-g) & \qquad D &= D(f-g) \end{aligned} \tag{46}$$

  Equation (25)and (26) then gives

$$C = \gamma_1 \cdot (f-g) \qquad\qquad D = \gamma_2 \cdot (f-g) \tag{47}$$

  for some constants γ_1, γ_2 and (23), (24) can be solved by

$$X = -\log\left[\frac{\gamma_1^2(a+b)^2}{8c_1^2(2+\delta b^2-\delta ab)}(f+g)\cosh^2\left(c_1 \log\frac{c_2}{f+g}\right)\right] \tag{48}$$

$$Y = -\log\left[\frac{\gamma_2^2(a+b)^2}{8c_1^2(2+\delta a^2-\delta ab)}(f+g)\cosh^2\left(c_1 \log\frac{c_2}{f+g}\right)\right] \tag{49}$$

  where c_1, c_2 are the integration constants. Without loss of generality one can take $c_1, c_2 > 0$. One can then integrate (28) and (29) to have

$$S = -\frac{4+\delta(a-b)^2}{(a+b)^2}\left[(4c_1^2+1)\log(f+g) + 4\log\cosh\left(c_1 \log\frac{c_2}{f+g}\right)\right] \tag{50}$$

and then

$$\begin{aligned} e^{-M} = &\left[\frac{\gamma_1^2(a+b)^2}{8c_1^2(2+\delta b^2-\delta ab)}\right]^{\frac{b}{a+b}\eta}\left[\frac{\gamma_2^2(a+b)^2}{8c_1^2(2+\delta a^2-\delta ab)}\right]^{\frac{-a}{a+b}\eta} f_u g_v (f+g)^{-(1-\delta)+(4c_1^2+1)\frac{4+\delta(a-b)^2}{(a+b)^2}+\frac{b-a}{a+b}\eta} \\ &\times\left[\cosh\left(c_1\log\frac{c_2}{f+g}\right)\right]^{\frac{2(b-a)}{a+b}\eta+\frac{4\left[4+\delta(a-b)^2\right]}{(a+b)^2}} \end{aligned} \tag{51}$$

The other components of the metric are

$$e^{nA} = \left[\frac{\gamma_1^2(a+b)^2}{8c_1^2(2+\delta b^2-\delta ab)}\right]^{\frac{b}{a+b}}\left[\frac{\gamma_2^2(a+b)^2}{8c_1^2(2+\delta a^2-\delta ab)}\right]^{\frac{-a}{a+b}}(f+g)^{\frac{2b}{a+b}}\left[\cosh^2\left(c_1\log\frac{c_2}{f+g}\right)\right]^{\frac{b-a}{a+b}} \tag{52}$$

$$e^{mB} = \left[\frac{\gamma_1^2(a+b)^2}{8c_1^2(2+\delta b^2-\delta ab)}\right]^{\frac{-b}{a+b}}\left[\frac{\gamma_2^2(a+b)^2}{8c_1^2(2+\delta a^2-\delta ab)}\right]^{\frac{a}{a+b}}(f+g)^{\frac{2a}{a+b}}\left[\cosh^2\left(c_1\log\frac{c_2}{f+g}\right)\right]^{\frac{a-b}{a+b}} \tag{53}$$

and the dilaton field is

$$e^{\phi} = \left[\frac{\gamma_1^2(a+b)^2}{8c_1^2(2+\delta b^2-\delta ab)}\right]^{\frac{-1}{a+b}}\left[\frac{\gamma_2^2(a+b)^2}{8c_1^2(2+\delta a^2-\delta ab)}\right]^{\frac{-1}{a+b}}(f+g)^{\frac{-2}{a+b}}\left[\cosh^2\left(c_1\log\frac{c_2}{f+g}\right)\right]^{\frac{-2}{a+b}} \tag{54}$$

The above solutions give a four-parameters family of solution labelled by $(\gamma_1, \gamma_2, c_1, c_2)$ in the F-region. In the same way, they reduce to the solutions in the L-region, R-region and in the P-region if one do the following replacements

$$f(u) = f_0 \qquad f_u|_{f_0} = -1 \qquad \text{for} \quad u < 0 \tag{55}$$

$$g(v) = g_0 \qquad g_v|_{g_0} = -1 \qquad \text{for} \quad v < 0 \tag{56}$$

A key point here is that the conditions on f_u, g_v are different from the ones in $(pqrw)$-type solutions at the junction. This is due to the fact that at the junction the only possible singular part in the metric arises only from f_u, g_v in e^{-M}. As we shall see, this fact will restrict the near-junction expansion of f and g strictly. Similarly, the patching of the solutions will put constraints on the parameters $\gamma_1, \gamma_2, c_1, c_2$.

2.2 Two-flux-CPW solutions when $a = -b$

In the case that $a = -b$, (32) is singular and one needs to change the variables as follows

$$X = \phi + \delta a V \qquad Y = a\phi - V \tag{57}$$

Then, in terms of (f, g) coordinates, we have

$$2C_{fg} + Y_f C_g + Y_g C_f = 0 \tag{58}$$

$$2D_{fg} - Y_f D_g - Y_g D_f = 0 \tag{59}$$

$$(f+g)X_{fg} + \frac{1}{2}(X_f + X_g) = 0 \tag{60}$$

$$(f+g)Y_{fg}+\frac{1}{2}\left(Y_f+Y_g\right)=\frac{1+\delta a^2}{4\delta}\left(e^Y C_f C_g-e^{-Y}D_f D_g\right) \tag{61}$$

and also

$$S_f+\frac{1}{2}e^Y C_f^2+\frac{1}{2}e^{-Y}D_f^2+\frac{\delta}{1+a^2\delta}(f+g)Y_f^2+\frac{1}{1+a^2\delta}(f+g)X_f^2=0 \tag{62}$$

$$S_f+\frac{1}{2}e^Y C_g^2+\frac{1}{2}e^{-Y}D_g^2+\frac{\delta}{1+a^2\delta}(f+g)Y_g^2+\frac{1}{1+a^2\delta}(f+g)X_g^2=0 \tag{63}$$

where

$$S=M-(1-\delta)\log(f+g)+\log(f_u g_v)+\eta V \tag{64}$$

A special case with $a=-b$ is that $a=b=0$. Then the theory is not a dilatonic gravity any more and reduces to a gravity theory with a n-form and a m-form potential. The CPW solutions of such a theory haven't been discussed before, as far as we know.

Let us first consider the equation on X. Note that it takes the same form as in the standard pure gravitational plane wave collision, and it can be solved by the Khan-Penrose-Szekeres solution:

$$X=\kappa_1\log\frac{w-p}{w+p}+\kappa_2\log\frac{r-q}{r+q} \tag{65}$$

where κ_1 and κ_2 are integration constants.

- $(pqrw)$-type solution:

  We may wish that there exit the same kind of $(pqrw)$-type solution, even though the set of coupled differential equations on (C,D,Y) looks more involved than the one flux case. Our ansatz are the following:

$$Y=-\log c_1\frac{rw+pq}{rw-pq} \tag{66}$$

$$C=\gamma_1(pw-rq) \tag{67}$$

$$D=\gamma_2(pw+rq) \tag{68}$$

  which solves (58) and (59) automatically and from (61) we find that

$$c_1\gamma_1^2+\frac{\gamma_2^2}{c_1}=\frac{8\delta}{\alpha} \tag{69}$$

  where

$$\alpha=1+\delta a^2 \tag{70}$$

  and c_1 is a constant. After integrating (62) and (63) with X given by (65), we obtain

$$\begin{aligned}S=b_1\log(1-2f)(1+2g)+b_2\log(1+2f)(1-2g)+(b_3-1+\delta)\log(f+g)+\\+\frac{2\kappa_1\kappa_2}{\alpha}\log\left(\frac{1}{2}+2fg+2pqrw\right)\end{aligned} \tag{71}$$

  Where

$$b_1=\frac{\kappa_1^2+\delta}{\alpha},\qquad b_2=\frac{\kappa_2^2+\delta}{\alpha},\qquad b_3=1-\delta-\frac{\delta+(\kappa_1+\kappa_2)^2}{\alpha} \tag{72}$$

  The components of the metric are given by

$$\begin{aligned}e^{-M}&=c_1^{\frac{\eta}{\alpha}}f_u g_v[(1-2f)(1+2g)]^{-b_1}[(1+2f)(1-2g)]^{-b_2}(f+g)^{-b_3}\times\\&\times\left[\frac{1}{2}+2fg+2pqrw\right]^{-\frac{2\kappa_1\kappa_2}{\alpha}}\left(\frac{rw+pq}{rw-pq}\right)^{\frac{\eta}{\alpha}}\left[\left(\frac{w-p}{w+p}\right)^{\kappa_1}\left(\frac{r-q}{r+q}\right)^{\kappa_2}\right]^{\frac{a\eta}{\alpha}}\end{aligned} \tag{73}$$

$$e^{nA} = c_1^{\frac{1}{\alpha}}(f+g)\left(\frac{rw+pq}{rw-pq}\right)^{\frac{1}{\alpha}}\left[\left(\frac{w-p}{w+p}\right)^{\kappa_1}\left(\frac{r-q}{r+q}\right)^{\kappa_2}\right]^{\frac{a}{\alpha}} \tag{74}$$

$$e^{mB} = c_1^{\frac{-1}{\alpha}}(f+g)\left(\frac{rw+pq}{rw-pq}\right)^{\frac{-1}{\alpha}}\left[\left(\frac{w-p}{w+p}\right)^{\kappa_1}\left(\frac{r-q}{r+q}\right)^{\kappa_2}\right]^{\frac{-a}{\alpha}} \tag{75}$$

$$e^{\phi} = \left(\frac{rw+pq}{rw-pq}\right)^{-\frac{\delta a}{\alpha}}\left[\left(\frac{w-p}{w+p}\right)^{\kappa_1}\left(\frac{r-q}{r+q}\right)^{\kappa_2}\right]^{\frac{1}{\alpha}} \tag{76}$$

Actually, taking into account the continuous condition on the metric, we may fix the value of $c_1 = 1$ and the above solutions take exactly the same form as the $(pqrw)$-type flux-CPW solution in [13]. So finally we have a three-parameter solutions labelled by $(\gamma_1(\text{or}\gamma_2), \kappa_1, \kappa_2)$.

- $(f \pm g)$-type solution:

  Here, we may use the same ansatz (45) and have

$$C = \gamma_1 \cdot (f-g) \qquad\qquad D = \gamma_2 \cdot (f-g) \tag{77}$$

with γ_1, γ_2 being constants. Now the equation on Y reduces to

$$(f+g)Y_{fg} + \frac{1}{2}(Y_f + Y_g) = -\frac{1+\delta a^2}{4\delta}\left(\gamma_1^2 e^{Y} - \gamma_2^2 e^{-Y}\right). \tag{78}$$

Unfortunately, we are not able to solve the above equation analytically. We will not try to analyze this kind of solution in the following discussion.

3 Junction conditions and future singularity

3.1 Junction conditions on the metric

The junction conditions play an important role in the discussion on CPW. Once one has the CPW solutions in the different regions, one needs to paste these solutions together. Besides the usual continuous and invertible conditions on the metric, one has to impose some kind of junction conditions on the metric to get an acceptable physical solution. More precisely, under the natural requirement that the stress tensor could be piecewise continuous instead of being continuous, namely the stress tensor may have finite jump but not δ-function jump across the junction, the Ricci tensor is allowed to be piecewise continuous. This leads to following junction condition on the metric[1]:

1. If the metric is C^1, then impose the Lichnerowicz condition: the metric has to be at least C^2. Otherwise, if the metric is piecewise C^1, then impose the OS junction conditions[14] which require :

$$g_{\mu\nu}, \qquad \sum_{ij} g^{ij} g_{ij,0}, \qquad \sum_{ij} g^{i0} g_{ij,0}, \qquad (i, j \neq 0). \tag{79}$$

to be continuous across the null surface (note that "0" in the above formulae stands for $u = 0$ or $v = 0$). From our ansatz on the metric, the OS condition means that U, V, M need to be continuous and $U_u = 0$ across the junction at $u = 0$. The same happens at the junction $v = 0$.

However, the above Lichnerowicz/OS condition on the metric is not enough. To be physically sensible, the curvature invariants R and R_2 should not have poles at the junction, namely

2. The curvature invariants R, R_2 do not blow up at the junction.

Usually, when discussing the CPW solutions, one does not put on any constraints on $R_{\mu\nu\alpha\beta}, R_4$ or other higher curvature invariants. We shall not discuss this issue neither.

[1] For a detailed discussion on the junction condition, please see [13, 14].

In the last section, we have constructed the two-flux-CPW solutions to the equations of motions, we now apply the junction conditions to these solutions. Before that, let us assume that the near-junction expansion of $f(u \geq 0)$ and $g(v \geq 0)$ take the form:

$$f = f_0(1 - d_1 u^{n_1}) \qquad u \sim 0^+ \tag{80}$$

$$g = g_0(1 - d_2 v^{n_2}) \qquad v \sim 0^+ \tag{81}$$

The boundary exponents n_i's indicate the behavior of the metric near the junction.

3.2 Imposing junction conditions on the $(pqrw)$-type solution

- Lichnerowicz/O'Brien-Synge junction conditions

In the general case that $a \neq -b$, from the continuous and invertible condition on the metric components we have

$$f_0 = g_0 = 1/2. \tag{82}$$

$$c_1^{\frac{b}{a+b}\eta} c_2^{\frac{-a}{a+b}\eta} = 1 \Rightarrow c_1 = c_2^{\frac{a}{b}} \tag{83}$$

from e^A and e^B. As for the continuity of e^{-M}, the condition (42) requires

$$\rho = 1 - \frac{1}{n_i}, \qquad d_i = \left(\frac{2}{n_i}\right)^{n_i}, \qquad i = 1, 2. \tag{84}$$

Let us zoom in the behavior of the metric near $u \sim 0, v \sim 0$. Actually it is enough to focus on $u \sim 0$ since the analysis for $v \sim 0$ is very similar. For $(pqrw)$-type solution, we have for $u \sim 0$

$$U_u = \left(u^{n_1-1}\frac{-d_1 n_1}{1+2g} + l.s.t.\right)\Theta(u) \tag{85}$$

$$V_u = \left(u^{\frac{n_1}{2}-1}e_1(\nu) + l.s.t.\right)\Theta(u) \tag{86}$$

$$nA_u = \left(u^{\frac{n_1}{2}-1}e_1(\nu) + l.s.t.\right)\Theta(u) \tag{87}$$

$$mB_u = \left(-u^{\frac{n_1}{2}-1}e_1(\nu) + l.s.t.\right)\Theta(u) \tag{88}$$

$$M_u = \left(-\eta u^{\frac{n_1}{2}-1}e_1(\nu) + l.s.t.\right)\Theta(u) \tag{89}$$

where l.s.t. in the above stands for less singular terms and $e_0(v), e_1(v)$ are some nonzero functions of v. It is easy to see that the metric is C^1 if $n_1 > 2$ and is piecewise C^1 if $n_1 \leq 2$. For the case that metric is C^1, the Lichnerowicz condition is satisfied. As for the case that the metric is piecewise C^1, when $n_1 \leq 2$, we need to impose the OS junction conditions, which require that U_u to be continuous (i.e. equal to zero) across the junction at u=0. This leads to the constraint

$$1 < n_i \leq 2. \tag{90}$$

In the special case that $a = -b$, the continuous and invertible condition on the metric tell us that

$$c_1 = 1, \qquad \gamma_1^2 + \gamma_2^2 = \frac{8\delta}{\alpha} \tag{91}$$

and

$$b_i = 1 - \frac{1}{n_i}, \qquad d_i = \left(\frac{2}{n_i}\right)^{n_i} \tag{92}$$

where $i = 1, 2$. With $c_1 = 1$, we find that the $(pqrw)$-type two-flux-CPW solutions take the similar from as the one-flux $(pqrw)$-type solution in [13]. The analysis of the behavor near $u \sim 0$ gives the same constraints.

Therefore, after imposing the Lichnerowicz or the O'Brien-Synge junction conditions, we have the following allowed possibilities

$$\begin{cases} (1) \;\; 1 < n_i \leq 2 & \text{metric is piecewise } C^1 \\ (2) \;\; n_i > 2 & \text{metric is at least piecewise } C^2 \end{cases} \tag{93}$$

in the $(pqrw)$-type solutions.

- On R,R_2

As we have discussed, even though the Ricci tensor has no δ-function jump at the junction, one has to be careful to keep it from blowing up at the junction. Instead of studying the Ricci tensor, equivalently one can investigate the behavior of the curvature invariants R, R_2 at the junctions. Requiring the absence of blow-up at the junction put more constraints on the n_i's. From our metric, we know that the scalar curvature has the form

$$R = 2e^M R_{uv} + ne^{-A} R_{xx} + me^{-B} R_{yy} \tag{94}$$

From the explicit expression of R_{uv}, R_{xx} and R_{yy}, we know that the the singularity behavior of R is controlled by U_u, V_u, M_u. And V_u is the most singular object which tell us that R is non-singular at the boundary if $n_i \geq 2$

Equivalently, we can use the equations of motions to find the following simple form for R

$$R = 2e^M \phi_u \phi_v + \frac{m-n}{m+n} \frac{e^{M+X}}{f+g} C_u C_v + \frac{n-m}{m+n} \frac{e^{M+Y}}{f+g} D_u D_v \tag{95}$$

The analysis on the boundary behavior for the field ϕ, C and D gives us the same answer, namely, in order for R not to blow up at the junction, one requires that $n_i \geq 2$.

The careful discussion on the R_2, which is of the form

$$R_2 = 2e^{2M} R_{uv}^2 + 2e^{2M} R_{uu} R_{vv} + ne^{-2A} R_{xx}^2 + me^{-2B} R_{yy}^2, \tag{96}$$

impose the same constraints on n_i's.

In summary, after taking into account the constraints from all the junction conditions, we find the following physical possibilities :

$$\rho = 1 - \frac{1}{n_i}, \qquad \text{for} \quad a \neq -b \tag{97}$$

$$b_i = 1 - \frac{1}{n_i}, \qquad \text{for} \quad a = -b \tag{98}$$

but with the same constraints on the boundary exponents:

$$\begin{cases} (1) \; n_i = 2 & \text{metric is piecewise } C^1 \\ (2) \; n_i > 2 & \text{metric is at least piecewise } C^2 \end{cases} \tag{99}$$

on the $(pqrw)$-type solutions.

3.3 Imposing junction conditions on $(f \pm g)$-type solutions

Here, we will only discuss the case when $a \neq -b$. The continuity of e^A and e^B is automatic. As for the continuity of e^{-M}, the condition (54) requires

$$n_1 = 1 \qquad\qquad d_1 = 2 \tag{100}$$

If one fixes the normalization of the metric such that A=B=M=0 in the P-region, then we get

$$\left[\frac{\gamma_1^2(a+b)^2}{8c_1^2(2+\delta b^2-\delta ab)}\right]^{\frac{b}{a+b}}\left[\frac{\gamma_2^2(a+b)^2}{8c_1^2(2+\delta a^2-\delta ab)}\right]^{\frac{-a}{a+b}}\left[\cosh^2\left(c_1\log\frac{c_2}{f+g}\right)\right]^{\frac{b-a}{a+b}} = 1 \tag{101}$$

$$\left[\frac{\gamma_1^2(a+b)^2}{8c_1^2(2+\delta b^2-\delta ab)}\right]^{\frac{b}{a+b}\eta}\left[\frac{\gamma_2^2(a+b)^2}{8c_1^2(2+\delta b^2-\delta ab)}\right]^{\frac{-a}{a+b}\eta}\left[\cosh^2\left(c_1\log\frac{c_2}{f+g}\right)\right]^{\frac{b-a}{a+b}\eta+\frac{2\left[4+\delta(a-b)^2\right]}{(a+b)^2}} = 1 \tag{102}$$

One solution of these constrains is

$$c_2 = 1 \tag{103}$$

$$c_1^2 = \left[\frac{\gamma_1^2(a+b)^2}{8(2+\delta b^2-\delta ab)}\right]^{\frac{b}{b-a}}\left[\frac{\gamma_2^2(a+b)^2}{8(2+\delta a^2-\delta ab)}\right]^{\frac{-a}{b-a}} \tag{104}$$

Similarly, for $u \sim 0$ we have the following expansion

$$U_u = \frac{-2}{1+2g}\Theta(u) \tag{105}$$

$$V_u = \left[\frac{2(a-b)}{(a+b)(1+2g)} + \frac{a-b}{a+b}e_1(v)\right]\Theta(u) \tag{106}$$

$$nA_u = \left[\frac{-4b}{(a+b)(1+2g)} + \frac{a-b}{a+b}e_1(v)\right]\Theta(u) \tag{107}$$

$$mB_u = \left[\frac{4a}{(a+b)(1+2g)} + \frac{a-b}{a+b}e_1(v)\right]\Theta(u) \tag{108}$$

$$M_u = \left[\frac{2b_0}{1+2g} - \left(\frac{2\left(4+\delta(a-b)^2\right)}{(a+b)^2} - \frac{\eta(a-b)}{a+b}\right)e_1(v)\right]\Theta(u) \tag{109}$$

Where

$$b_0 = -(1-\delta) + (4c_1^2+1)\frac{4+\delta(a-b)^2}{(a+b)^2} - \frac{\eta(a-b)}{a+b} \tag{110}$$

$$e_1(v) = 2\tanh\left(c_1\log\frac{2}{1+2g}\right)\frac{2c_1}{1+2g} \tag{111}$$

The key point here is that the continuous condition on the metric requires (55,56), which fix the boundary exponents completely:

$$n_i = 1 \qquad\qquad i = 1, 2. \tag{112}$$

This means that the metric could only be piecewise C^1. The further imposition of OS condition requires that U_u is continuous. However, the above expansion U_u is proportional to the step function, showing the violation of the OS condition. Therefore, the $(f \pm g)$-type solution is unphysical!

3.4 Future singularity of the solution

In this subsection we will see if the future curvature singularity will generically appear in our new higher dimensional flux-CPW solutions. We will focus on the $(pqrw)$-type solutions, which are the only physical solutions we have.

First we define a hyper-surface S_0:

$$f(u)+g(v)=0 \tag{113}$$

near which the metric may blow up or vanish. Near S_0 we have

$$\frac{rw+pq}{rw-pq}\sim(f+g)^{-1} \tag{114}$$

and

$$\frac{w-p}{w+p}\sim(f+g),\qquad \frac{r-q}{r+q}\sim(f+g) \tag{115}$$

When $a\neq -b$, the singular behavior of the $(pqrw)$-type solution near S_0 read:

$$\begin{aligned}
e^{-M} &\sim (f+g)^{-\left[1-\delta-\frac{4+\delta(a-b)^2}{(a+b)^2}-\frac{a-b}{a+b}\eta\right]} && (116)\\
e^{nA} &\sim (f+g)^{\frac{2a}{a+b}} && (117)\\
e^{mB} &\sim (f+g)^{\frac{2b}{a+b}} && (118)\\
e^{\phi} &\sim (f+g)^{\frac{2}{a+b}} && (119)
\end{aligned}$$

The regularity and invertibility of the metric asks the exponents on the R.H.S to vanish simultaneously. It is obvious that this is impossible. Therefore at $f+g=0$ the metric is singular. On the other hand, the singularity of the metric could be just a coordinate singularity. To check if the curvature singularity do appear, let us turn to the curvature invariants R, R_2 and R_4. Note that, near S_0 we have

$$\frac{\partial^{l_1+l_2}M}{\partial u^{l_1}\partial v^{l_2}}\sim\frac{\partial^{l_1+l_2}A}{\partial u^{l_1}\partial v^{l_2}}\sim\frac{\partial^{l_1+l_2}B}{\partial u^{l_1}\partial v^{l_2}}\sim(f+g)^{-(l_1+l_2)} \tag{120}$$

Then from the expressions of the Ricci tensor and Riemann tensor listed in the appendix, it is easy to check that the most singular terms near S_0 in R^2 , R_2 and R_4 are all taking the following generic form

$$e^{2M}(f+g)^{-4}\sim(f+g)^{-2\left(1+\delta+\frac{4+\delta(a-b)^2}{(a+b)^2}+\frac{a-b}{a+b}\right)} \tag{121}$$

The exponent is

$$-2\left[1+\delta\left(1-\frac{\eta^2}{4\delta^2}\right)+\delta\left(\frac{a-b)}{a+b}+\frac{\eta}{2\delta}\right)^2+\frac{4}{(a+b)^2}\right]<0 \tag{122}$$

Therefore the $(pqrw)$-type solutions for $a\neq -b$ will always develop a late time curvature singularity. So the curvature singularity will always be developed.

When $a=-b$, the singular behavior of the metric components near S_0 read:

$$\begin{aligned}
e^{-M} &\sim (f+g)^{-b_3-\frac{\eta}{\alpha}(1-a(\kappa_1+\kappa_2))} && (123)\\
e^{nA} &\sim (f+g)^{1-\frac{1}{\alpha}(1-a(\kappa_1+\kappa_2))} && (124)\\
e^{mB} &\sim (f+g)^{1+\frac{1}{\alpha}(1-a(\kappa_1+\kappa_2))} && (125)\\
e^{\phi} &\sim (f+g)^{\frac{1}{\alpha}(\delta a+(\kappa_1+\kappa_2))}. && (126)
\end{aligned}$$

Obviously, the exponents above cannot be vanishing at the same time, indicating the metric is singular at $(f+g)$. And similarly the singular behavior of R^2, R_2, R_4 near S_0 is dominated by

$$e^{2M}(f+g)^{-4}\sim(f+g)^{2[b_3+\frac{\eta}{\alpha}(1-a(\kappa_1+\kappa_2))-2]} \tag{127}$$

The exponent above could be shown to be less than -1, taking into account of the explicit value of η,α,b_3 and δ. This fact shows that the singularity is destined to be developed in the future.

4 Conclusions and Discussions

In this paper, we investigate the colliding plane wave solutions in a higher dimensional dilatonic gravity with two complementary fluxes, generalize the discussion on flux-CPW solutions in [13]. We manage to solve the equations of motions, which are more complicated than the ones in one flux case. Quite similarly, using two different ansatz we find two types of CPW solutions : $(pqrw)$-type and $(f \pm g)$-type. The $(pqrw)$-type solutions satisfy the Lichnerowicz/O'Brien-Synge junction conditions and could also keep the curvature invariants from blowing up at the junction. Precisely speaking, when $a \neq -b$, we have an one-parameter family of $(pqrw)$-type solutions, and when $a = -b$ we have a three-parameter family one.

Unfortunately, the $(f \pm g)$-type solutions break the junction conditions and are physically unacceptable. This fact may indicates that the two-flux background restrict the system more severe than the one-flux background. Recall the set of coupled differential equations on (X, Y, C, D). Comparing with the corresponding ones in one flux case, it is obvious that the equations here is more restrictive and harder to find the solutions. More crucially, in one flux case, the equation on X takes the same form as the standard one in pure gravitational colliding plane wave, which is related to Backlund transformation and inverse scattering method. It has Khan-Penrose-Szekeres solution whose implications in the metric is important as emphasized in [13]. In the two complementary fluxes case, technically we are short of this kind of solution. One interesting question is that if we relax the complementary condition, and take a more general ansatz on the metric, could we find less restrictive solutions[15]?

We have also shown that the physical two-flux $(pqrw)$-type solutions will always develop a late time curvature singularity, in consistent with the result in one-flux case. This may reflect the fact that the strong focus effect of gravity is universal, even in the higher dimensional gravity theories.

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A Riemann and Ricci tensors

In the paper, we make the following ansatz to the metric

$$ds^2 = 2e^{-M} du dv + e^A \sum_{i=1}^{n} dx_i^2 + e^B \sum_{j-1}^{m} dy_j^2 \tag{128}$$

with the functions M, A, B being functions of u, v. We have Ricci tensor

$$\begin{aligned}
R_{uu} &= -\frac{1}{2}\left[nA_{uu} + mB_{uu} + nM_uA_u + mM_uB_u + \frac{1}{2}(nA_u^2 + mB_u^2)\right] && (129)\\
R_{vv} &= -\frac{1}{2}\left[nA_{vv} + mB_{vv} + nM_vA_v + mM_vB_v + \frac{1}{2}(nA_v^2 + mB_v^2)\right] && (130)\\
R_{uv} &= M_{uv} - \frac{n}{2}A_{uv} - \frac{m}{2}B_{uv} - \frac{1}{4}(nA_uA_v + mB_uB_v) && (131)\\
R_{xx} &= -\frac{1}{2}e^{M+A}\left[2A_{uv} + nA_uA_v + \frac{m}{2}(A_uB_v + A_vB_u)\right] && (132)\\
R_{yy} &= -\frac{1}{2}e^{M+B}\left[2B_{uv} + nB_uB_v + \frac{n}{2}(A_uB_v + A_vB_u)\right] && (133)
\end{aligned}$$

where $x = x_i$ with $i = 1, \cdots, n$ and $y = y_j$ with $j = 1, \cdots, m$. And also we have the independent non-vanishing components of the Riemann tensor as following:

$$R_{uvuv} = -e^{M} M_{uv} \quad (134)$$

$$R_{xyxy} = -\frac{1}{4}e^{M+A+B}(A_u B_v + A_v B_u) \quad (135)$$

$$R_{uxvx} = -e^{A}(\frac{1}{2}A_{uv} + \frac{1}{4}A_u A_v) \quad (136)$$

$$R_{vxvx} = -e^{A}(\frac{1}{2}A_{vv} + \frac{1}{2}M_v A_v + \frac{1}{4}A_v^2) \quad (137)$$

$$R_{uxux} = -e^{A}(\frac{1}{2}A_{uu} + \frac{1}{2}M_u A_u + \frac{1}{4}A_u^2) \quad (138)$$

$$R_{uyvy} = -e^{B}(\frac{1}{2}B_{uv} + \frac{1}{4}B_u B_v) \quad (139)$$

$$R_{vyvy} = -e^{B}(\frac{1}{2}B_{vv} + \frac{1}{2}M_v B_v + \frac{1}{4}B_v^2) \quad (140)$$

$$R_{uyuy} = -e^{B}(\frac{1}{2}B_{uu} + \frac{1}{2}M_u B_u + \frac{1}{4}B_u^2). \quad (141)$$

References

[1] P. Szekeres, *Colliding gravitational waves*, Nature **228** (1970)1183.

[2] K. A. Khan and R. Penrose, *Scattering of Two Impulsive Gravitational Waves*, Nature **229** (1971)185.

[3] J. B. Griffiths, *Colliding Plane Waves in General Relativity*, Clarendon Press 1991, Oxford.

[4] F. J. Tipler, *Singularities From Colliding Plane Gravitational Waves*, Phys. Rev. D**22** (1980)2929.

[5] U. Yurtsever, *Instability of Killing-Cauchy Horizons in Plane-symmetric Spacetimes*, Phys. Rev. D**36** (1987)1662.
Structure of the Singularities Produced by Colliding Plane Waves, Phys. Rev. D**38** (1988)1706.

[6] A. Feinstein, K. E. Kunze and M. A. Vazquez-Mozo, *Initial Conditions and The Structure of the Singularity in Pre-big-bang Cosmology*, Class. Quantum Grav. **17** (2000)3599.
Entropy Generation and Inflation in Wave Collision Induced Pre-big-bang Cosmology, [hep-th/0004094].

[7] H. Kodama, *Uniqueness and Stability of Higher-Dimensional Black Holes*, [hep-th/0403030].

[8] R. Emparan and H. S. Reall, *A Rotating Black Ring Solution in Five-Dimensions*, Phys. Rev. Lett. **88** (2002)101101, [hep-th/0110260].

[9] R. Guven, Phys. Lett. B**191** (1987)275. D. Amati and C. Klimcik, *Nonperturbative Computation of the Weyl Anomaly for a Class of Nontrivial Backgrounds* , Phys. Lett. B**219** (1989)443.
G. Horowitz and A. Steif, *Spacetime Singularities in String Theory*, Phys. Rev. Lett. **64** (1990)260.

[10] M. Gurses and E. Sermutlu, *Colliding Gravitational Plane Waves in Dilaton Gravity*, Phys. Rev. D**52** (1995)809.
M. Gurses, E. O. Kahya and A. Karasu, *Higher Dimensional Metrics of Colliding Gravitational Plane Waves*, Phys. Rev. D**66** (2002)024029.
M. Gurses, Y. Ipekoglu, A. Karasu and C. Senturk, *Higher Dimensional Bell-Szekeres Metric*, Phys. Rev. D**68** (2003)084007, [gr-qc/0305105].

[11] M. Gutperle and B. Pioline, *Type IIB Colliding Plane Waves*, JHEP **0309** (2003)061, [hep-th/0308167].

[12] P. Bell and P. Szekeres, *Interacting Electromagnetic Shock Waves in General Relativity*, Gen. Rel. Grav. **5** (1974)275.

[13] B. Chen, C. S. Chu, K. Furuta and F. L. Lin, *Colliding plane waves in string theory*, JHEP **0402** (2004)020, [hep-th/0311135].

[14] S. O'Brein and J. L. Synge, *Jump conditions at discontinuities in general relativity*, Comm. Dublin Inst. Adv. Stud. A**9** (1952).

[15] B. Chen, *Colliding Plane Wave Solutions in String theory Revisited*, JHEP **0412** (2004)016, [hep-th/0409023].